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*Norwegian Geotechnical Institute*

Internal Report

Interpretation of Salt Concentrations in  
the Scandinavian Marine Clays as a Tool  
in Geotechnical Engineering.

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## I. INTRODUCTION

The purpose of this study is to demonstrate that an interpretation of the salt concentrations in the Scandinavian marine clays may yield valuable information about some of the geotechnical properties of these deposits.

The method of interpretation is based on elementary concepts of sea-salt diffusion superimposed on porewater flow. These concepts are developed into a general program for analysing one-dimensional geometries.

Three clay deposits are interpreted by these concepts. The clay deposits demonstrate the applicability of the method of interpretation and indicates the range of variation in diffusion constants to be expected in the Scandinavian marine clays. The experience with these deposits is developed into a general regional prediction of the maximum salt concentrations to be expected in the marine clay deposits of the inner Oslofjord area.

The information obtained by interpreting the salt concentrations of the marine clays may find the following geotechnical applications:

1. In a given deposit, free draining layers may be detected, the vertical velocity of flow may be determined, natural hydraulic conditions may be understood qualitatively, and salt concentrations may be predicted by interpolation between measurements.
2. The interpretation may be used generally, to predict whether a clay deposit has salt or fresh porewater, from knowledge of its location, thickness, and general geological condition.

## II. THE METHOD OF INTERPRETATION OF SALT CONCENTRATION IN MARINE CLAYS

The salt concentrations considered range from the initial sea water concentration to the concentration at which the exchange of ions between the soil skeleton and the free porewater influences the total salt concentration in the free porewater markedly (Moum et al., 1971, Söderblom, 1969). In this range the present salt concentrations are developed by two physical processes. These are:

1. Salt transportation, relative to the free porewater, by diffusion.
2. Transportation of the free porewater through the soil skeleton by hydraulic flow.

Diffusion occurs because salt will move through water, with a flux which is proportional to the gradient of salt concentration. The constant of proportionality is the diffusion constant. The salt concentration in a given volume may be described as a function of time by equating the net flux of salt out of the volume element, to the rate of loss of salt with time. For one-dimensional geometries, under conditions of constant porewater flow and constant diffusion constants, it may be shown that the governing differential equation is:

$$D \frac{\delta^2 x}{\delta z^2} - v \frac{\delta x}{\delta z} = \frac{\delta x}{\delta t} \quad (\text{Heiberg, 1972})$$

where

- D is the diffusion constant in  $m^2/\text{year}$
- x is the salt concentration
- v is the velocity of free porewater in  $m/\text{year}$
- z is the vertical coordinate in metres
- t is the time in years

This equation is simplified by introducing:

1. The following dimensionless coordinates:

$$\psi = \frac{x}{x_0} \quad \text{the concentration factor. } x_0 \text{ is a reference concentration, normally the initial one.}$$

$$Z = \frac{z}{H} \quad \text{the distance factor. } H \text{ is some reference distance, commonly the thickness of the deposit.}$$

$$T = \frac{Dt}{H^2} \quad \text{the time factor}$$

$$V = \frac{vH}{D} \quad \text{the velocity factor}$$

2. The auxiliary function:

$$\psi' = \psi e^{-\frac{VZ}{2} + \frac{V^2 T}{4}} \quad \text{where } e \text{ is the base of the natural logarithm.}$$

The simplified equation is:

$$\frac{\delta^2 \psi'}{\delta Z^2} = \frac{\delta \psi'}{\delta T}$$

which is recognized as the equation governing one-dimensional heat flow in a homogeneous medium. This equation is well studied. Details of the solution procedures and references to pertinent literature is given in the appendix.

### III. ANALYSIS OF THREE CLAY DEPOSITS

The salt concentrations presently measured in clay deposits may be interpreted by the concepts outlined above. When these concepts are to be used for quantitative interpretations, it is often useful to know the initial salt concentrations, and the boundary concentrations with time. This is not always required as a given distribution of the salt concentrations may be consistent with only one set of initial and boundary conditions. In the analysis of the three case records presented below, it has been necessary to know the time elapsed since the conditions at the top boundary changed from salt to fresh. This necessity is due to a lack of knowledge of the value of the diffusion constant of the clays. The time elapsed since the top surface became fresh are estimated from the present rates of isostatic uplift (0.0025 m/year at Lødöse, Jangdal, 1971 and 0.0037 m/year at the other sites, Hafsten, 1960). The time curve of isostatic uplift shows that these rates have been approximately constant over the times involved.

For the deposit at Lødöse, the time elapsed since the top surface became fresh is taken to be the time elapsed since the ground surface rose above sea level. For the other two cases, the time elapsed is taken to be the time interval since the groundwater table rose above the sea. This assumption is arbitrary and may be discussed. The value of the diffusion constant obtained from Vaterland, Central Oslo, is strongly influenced by this assumption as the terrain is located at an elevation near sea level.

In computing times from the rates of isostatic uplift, it is assumed that clay compression has not occurred since the terrain rose above sea level.

#### 3.1 Drammen, Norway

The deposits in Drammen have been described by Bjerrum (1967). The location of the two borings to be interpreted below are shown in Tegn.007. These are referred to as the Sundland and the Danviks gate profiles. The

clay deposit at Sundland was modeled by two layers, one representing the upper plastic clay, and the other representing the lower lean clay. The diffusion constants of the two clays were assumed to be different. Measured pore pressure at Sundland showed that the porewater travelled downward. It is assumed in the analysis that this condition has prevailed since the top surface became fresh. In reality the present hydraulic gradients must have developed gradually as the total head in the clay never can have been less than in the sea. The above assumption will most probably not cause great errors, however, because the hydraulic gradients are small.

The analysis (Tegn.010) showed that the measured salt concentrations are consistent with sets of values of the velocity factor and of ratios of the diffusion constants of the upper layer to that of the lower layer. The relation between permissible ratios and velocity factors are shown on Tegn.010. It is probable that the ratio of diffusion constants is similar in value to the ratio of the porosities (approximately 1.3), because the relative area available for the flow of ions is determined by this ratio. The corresponding velocity factor is 4.5.

The soil profile at Danviks gate is shown on Tegn.009. The pore pressures at Danviks gate show that the water is moving upward. The profile is interpreted on Tegn.010. The profiles from Sundland and Danviks gate indicate that no important changes in the deposit has occurred between the two sites. It is therefore reasonable to assume that the diffusion constants in the upper plastic clay layer are nearly equal at the two sites. Then the present groundwater elevations may not be used to compute the times since the top surfaces became fresh. This leads to the assumption that the clay deposits have settled an equal amount at the two sites, or that the top of the clay became fresh, while the groundwater level stood higher than at present. The according results are:

Total settlements after the top surfaces became fresh, or equivalent change in groundwater level . . . . .	1.0 m
Years before present when the top surface at Sundland became fresh . . . . .	1350 years
Years before present when the top surface at Danviks gate became fresh . . . . .	430 "
Diffusion constant of the plastic clay . . . . .	0.017 m <sup>2</sup> /yr
Diffusion constant of the lean clay . . . . .	0.014 "
Average vertical permeability . . . . .	0.077 m/yr or $24 \times 10^{-8}$ cm/sec.

### 3.2 Central Oslo, Norway

Two areas within Central Oslo are examined below: The Vaterland area, and the Studenterlunden area. The location of the borings used for the interpretations are shown on Tegn.011.

The soil conditions at Vaterland are shown on Tegn.012. A top layer of clay, extending down to elevation -6.0 metres, is characterized by silt layers and occasional stains of oxidation. The base of this layer has a high undrained shear strength. Geologically, the top layer is interpreted as being slide debris. It rests on a layer of homogeneous clay.

The salt concentrations are measured indirectly by measuring electrical resistivity. The inverse of the resistivity, or the electrical conductivity is generally proportional to the salt concentration in the porewater (Söderblom, 1969). A total of six boring logs are shown on Tegn.013. It is seen that the resistivity increases sharply at the base of the top layer in the region near the river. This indicates that considerable flow has occurred in this layer near the river, where the hydraulic gradients have horizontal components. The rise in resistivity indicates therefore that the base of the top layer is free draining relative to clay above and below.

The resistivity logs away from the river are strikingly similar, between borings and between the top and the base of the borings. This confirms the pore-pressure measurements showing no hydraulic gradients of any significance in the area. The logs of electrical resistivities are averaged and converted to electrical conductivity. The profile of electrical conductivity is interpreted on Tegn.014. Assuming that the top surface became fresh when the present groundwater table rose above sea level gives the following results:

Time elapsed since the top and bottom surfaces became fresh . . .	270 years
Diffusion constant . . . . .	0.013 m <sup>2</sup> /yr

The soil conditions, the hydraulic conditions, and the records of clay compression at Studenterlunden are given by Eide & al. (1972). The clay at Studenterlunden is similar to the clay found in the base layer at Vaterland. The pore pressure measurements indicate that there are insignificant gradients in the top part of the deposit which is to be interpreted.



The soil condition is interpreted on Tegn.014. Assuming that the top surface became fresh when the present groundwater level rose above se level \* gives the following results :

Time elapsed since the top surface became fresh:	1300 years
Diffusion constant . . . . . :	0.014 m <sup>2</sup> /year

This neglects that settlements are occurring. If it is assumed that the measured settlement has occurred at the present rate since deposition, the resulting diffusion constant will have a low value of 0.006 m<sup>2</sup>/year. Eide et al. suggest that the settlements may be initiated recently (i.e. during one of the last centuries) as a result of changed drainage conditions.

### 3.3 Lödöse, Göta River Valley, Sweden

The clay deposit at Lödöse is described by Söderblom (1969). The deposit has a higher clay content and a higher porosity than the clays discussed above.

The development of the Lödöse profiles was discussed qualitatively by Söderblom. His suggestions are made quantitative on Tegn.015.

The interpretation of the profiles is based on the following assumptions :

1. The elevation at each boring may be determined from a slope towards the river of 1:55.
2. The top surface of the clay became fresh when the terrain rose above se level.
3. The diffusion constant is invariant with depth at each boring.

The profile at 57.7 metres from the river is most readily interpreted. The initial concentration is still intact near mid-depth. The top and the base of the profile may therefore be interpreted separately. The results of the interpretation are :

Time elapsed since the top surface became fresh	540 years
Time elapsed since the base became fresh	3400 "
Diffusion constant	0.012 m <sup>2</sup> /year
Velocity of pore water flow	0.001 m/year

\*) As the ground surface here is about +7 m above sea level, the diffusion constant computed is not very sensitive to this assumption.

A similar analysis may be performed for the profile at 106.8 metres from the river. As the initial salt concentrations are reduced throughout this deposit, it is necessary to analyse the upper and lower parts of the profile simultaneously. The results of interpretation are:

Ground elevation . . . . .	2.25 m a. s. l.
Time elapsed since the top surface became fresh	895 years
Time elapsed since the base surface became fresh	2100 "
Diffusion constant . . . . .	0.016 m <sup>2</sup> /year
Velocity of vertical flow . . . . .	0.004 m/year

For the profiles at 157.6 and 180.5 metres from the river it is justified to neglect that the two surfaces did not become fresh simultaneously because the present concentrations would not be markedly affected by these differences. The results of the interpretation of the profile at 157.6 m from the river are:

Ground elevation . . . . .	3.16 m a. s. l.
Time elapsed since the top surface became fresh	1265 years
Diffusion constant . . . . .	0.026 m <sup>2</sup> /year
Velocity of vertical flow . . . . .	0.007 m/year

The results for the profile at 180.5 m from the river are:

Ground elevation . . . . .	3.60 m a. s. l.
Time elapsed since the top surface became fresh	1430 years
Diffusion constant . . . . .	0.030 m <sup>2</sup> /year
Velocity of vertical flow . . . . .	0.009 m/year

The salt concentrations in the profile at 212.3 m from the river are too low for an interpretation to be made.

The profile at 2.0 m from the river is not interpreted quantitatively. The reason is that the salt concentrations are influenced by horizontal components of flow. This is most clearly seen at a depth of 25 metres. Here, there is a sharp local drop in conductivity. The same drop is observed at 57.7 metres from the river. This is indicative of a layer of lower water content than the clay. At 2.0 m from the river the conductivity exhibits a general depression around the layer. This indicates that the salt content in the layer is reduced by lateral flow and that diffusion is occurring towards this layer. Most probably the layer is a sand layer which is free draining relative to the clay. At

the top portion of the profile the salt concentrations are reduced as a result of the existing horizontal components of flow. There is no evidence of free draining layers here.

#### 3.4. A discussion of the value of the diffusion constant

The diffusion constant of the salts of sea water in the marine clays is an essential parameter. A best estimate of this parameter is derived from each of the three case records reviewed. The estimated values of the diffusion constants are plotted on Tegn.016 against porosity. An upper limit on the value of the diffusion constant is fixed at each porosity by the value of the diffusion constant of the salt in water. The diffusion constant of the salt in water is  $0.04 \text{ m}^2/\text{year}$ . At any porosity this maximum value is to be reduced because only a fraction of the total area, approximately equal to the porosity, is available for salt diffusion. The remaining area is occupied by the mineral particles. Tegn.016 shows how the diffusion constants derived from the case records are distributed.

There is some uncertainty connected with the determination of each value of the diffusion constant, as is clear from the reviews of the case records. Tegn.016 distinguishes between values that are considered well documented, and values that are not. The values from Lödöse are uncertain because the time elapsed since the top surface became fresh is not known precisely. All the values on Tegn.016 are derived on the assumption that no, or only minor compressions of the clay layers have occurred since the top surfaces became fresh. A compression of the clay would increase the computed times elapsed since the deposits rose above sea. The corresponding diffusion constants would be lower than those shown on Tegn.016.

Söderblom has computed diffusion constants from the sites of the large landslides at Surte and Göta in the Göta River valley. The initial condition is a discontinuity in salt concentration at the lowest sliding surface.

Four out of five determinations indicate diffusion constants much lower than those shown on Tegn.016, varying between  $0.003$  and  $0.005 \text{ m}^2/\text{year}$ . The fifth determination indicates a diffusion constant of  $0.019 \text{ m}^2/\text{year}$ , which is in good agreement with the "well documented" values on Tegn.019.

From the discussion above, it is clear that some ambiguity is present in the available data on the diffusion constant. It is therefore of interest to measure the magnitude of this constant under controlled conditions in the laboratory.

#### IV. A REGIONAL INTERPRETATION OF THE SALT CONCENTRATIONS IN MARINE CLAYS OF THE INNER OSLOFJORD AREA

Some general statements may be made regarding the salt concentrations to be expected in the marine clay deposits of the inner Oslofjord area.

Firstly, Tegn.018 shows how the maximum of the dimensionless salt concentration factor varies as a function of the dimensionless time and velocity factors, when the upper and lower boundaries of the deposit are made fresh simultaneously. The time elapsed since the top boundary became fresh may be estimated from the elevation of this surface on the basis of the known isostatic uplift which has occurred.

From a knowledge of the time elapsed since the surfaces became fresh, and the diffusion constant, it is possible to construct an upper limit of the salt concentration which occurs in any deposit as a function of the elevation of the top surface, the thickness of the deposit, and the fluid velocity. Tegn.017 shows a series of such upper limits.

#### V. GEOTECHNICAL ENGINEERING APPLICATIONS

The interpretation of salt concentration profiles may find several geotechnical engineering applications.

- Permeability determination: For particular deposits it may be possible to evaluate the bulk vertical permeability from field measurements of salt concentrations and hydraulic gradients. In regions where the hydraulic gradients have horizontal components, it may be possible to determine free draining layers. The case records of Drammen, Lødöse and Vaterland in Central Oslo illustrate this.

- Natural hydraulic conditions: Salt concentration profiles may be used to evaluate the natural pore water flow condition and thereby the natural pore pressure conditions. This is illustrated by the case records from Vaterland, Central Oslo and Lödöse.
  
- Corrosion conditions: A quantitative understanding of the salt transportation makes it possible to interpolate values of salt concentrations between borings in a given uniform horizontal deposit. As corrosion rates on steel piles depends strongly on the salt concentration, this may help predicting corrosion rates. For many practical cases, a regional interpretation of salt concentrations may be sufficient to determine whether there exists a corrosion problem or not.
  
- Quick clay occurrence: Quick clay may not occur in deposits where the salt concentration in the pore water is above a few grams per litre. A knowledge of the development of the salt concentration profiles may make it possible to evaluate the zones in a deposit where quick clay may not occur because of too high salt concentrations.

## SUMMARY AND CONCLUSIONS

The main points of this study are:

- The transportation of salt in the Scandinavian marine clay deposits is modeled by applying elementary concepts of diffusion, superimposed on pore water flow.
  
- The salt concentrations predicted by this model are remarkably consistent with the salt concentrations measured in the clay deposits.
  
- The known rates of isostatic uplift in Scandinavia have made it possible to estimate absolute values of the diffusion constants and permeabilities from measurements of salt concentration profiles and hydraulic gradients. There is some uncertainty connected with these values. The uncertainty arises mainly from the unknown amounts of clay compression which has occurred since the surface became fresh. The determinations of the values of the diffusion constants should therefore be supplemented by laboratory determinations. The determinations of the diffusion constant presently available indicates

that it varies within narrow limits, depending on the porosity.


The following conclusions may be drawn from these main points :

- (1) Examination of the salt concentrations in marine clay deposits yield reliable information on the flow conditions in the clays.
- (2) The salt concentration at any point in a horizontal deposit may be found by interpolation between measured points. Furthermore, the salt concentration in a deposit may be estimated in general terms from its location and geological setting.

These conclusions apply to the marine clay deposits where the salt concentration is still above a few grams per litre.

The conclusions, and the fact that salt concentrations of clays are easily measured, establish the method of interpretation of salt concentrations as a potential tool of geotechnical engineering.

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## APPENDIX

Description of the Diffusion of Salt in Porous Media  
with moving Pore Fluids and Users Manuals to  
Computer Programs

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The diffusion of salt in porous media with moving pore fluids will be described by its governing differential equation. Examples of solution procedures, both analytical and numerical, will be given. The corresponding computer program listings are presented together with users manuals to the programs.

The governing differential equation

The differential equation governing the transportation of salt in porous media with moving pore fluids is derived by Heiberg (1972). Five assumptions are required to develop the equation describing one-dimensional flow. These are:

- (1) The flux of salt ions in stationary pore fluid is proportional to the salt concentration gradient.
- (2) There are no sources or sinks of ions in the body.
- (3) The diffusion constant is invariable.
- (4) The fluid velocity is invariable.
- (5) The geometry is one-dimensional.

The appropriate differential equation may then be shown to be:

$$D \frac{\delta^2 x}{\delta z^2} - v \frac{\delta x}{\delta z} = \frac{\delta x}{\delta t} \quad (1)$$

where

D is the diffusion constant in  $m^2/\text{year}$

x is the salt concentration

z is the vertical coordinate in meters

v is the fluid velocity in  $m/\text{year}$

t is time in years

Equation (1) may be written in terms of dimensionless parameters:

$$\frac{\delta^2 \psi}{\delta z^2} - V \frac{\delta \psi}{\delta z} = \frac{\delta \psi}{\delta T} \quad (2)$$

where

$\psi = \frac{x}{x_0}$  the concentration factor.  $x_0$  is some reference concentration, usually the initial concentration.

$z = \frac{z}{H}$  the depth factor.  $H$  is some reference distance, usually the full thickness of the deposit.

$T = \frac{Dt}{H^2}$  the time factor.

$V = \frac{vH}{D}$  the velocity factor.

Equation (2) may be transformed to a more convenient form, introducing the variable:

$$\psi' = \psi e^{-\frac{vz}{2} + \frac{V^2 T}{4}} \quad (3)$$

where

$e$  is the base of the natural logarithm.

Equation (2) becomes:

$$\frac{\delta^2 \psi'}{\delta z^2} = \frac{\delta \psi'}{T}$$

which is recognized as the equation governing heat flow in a homogeneous medium. This equation is well studied (Hildebrand, 1962; Carslaw and Jaeger, 1959; Crandall, 1956; Zienkiewicz, 1971). The boundary and initial conditions of a problem described by equation 4 must be transformed according to equation (3). The solutions to this problem will therefore be dissimilar to the more common solutions of the heat flow equation. Analytical solution to equation (4) may be obtained for several classes of boundary and initial conditions. The equation may also be solved numerically for general initial and boundary conditions with about the same computational effort as is required for the analytical solutions, provided a computer is used. The main purpose of the analytical solutions is therefore presently to provide true values against which the accuracy of the approximate numerical values may be checked.



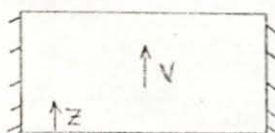
Analytical solutions

A common case is the one where the salt concentration in the deposit is constant initially, and where the top and bottom boundaries become fresh simultaneously and remain at constant concentrations with time. This case is solved by Heiberg (1972), using the method of separation of variables. The solution is:

$$\psi(z_1 T) = \sum_{n=0}^{\infty} \frac{2n\pi}{\frac{V^2}{4} + n^2 \pi^2} \left(1 - e^{-\frac{V}{2} T} \cdot (-1)^n\right) e^{-(n^2 \pi^2 + \frac{V^2}{4}) T + \frac{Vz}{2}} \sin n\pi z \quad (5)$$

Another common case is the one where the salt concentration is constant initially, at which time the top surface becomes fresh. The top surface remains fresh subsequently. The boundary condition at the base remains at the initial concentration. This problem may also represent the cases where the initial concentration is unchanged at some depth in the deposit.

The initial and boundary conditions may be written as:



$z = 1, \psi = 1, \psi' = e^{-\frac{V}{2} T + \frac{V^2 T}{4}} = F(T)$   
 $\psi_{\text{initial}} = 0; \psi'_{\text{initial}} = 0$   
 $z = 0, \psi = 0, \psi' = 0$

where the concentration factor for convenience is chosen to vary from -1 to 0.

The boundary condition imposed at the top surface on the function  $\psi'$  varies with time. The solution may be obtained by use of the superposition integral, as outlined by Hildebrand (1962, p. 451).

The effect of a unit change in  $\psi'$  at the top boundary is:

$$U(z_1 T) = z + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin n\pi z \cdot e^{-n^2 \pi^2 T} \quad (6)$$

Summing the effect of succeeding step variations of  $\psi'$  bounded by the function describing the boundary condition on  $\psi'$  and taking the limit as the duration of the steps go to zero:

$$\psi'(z_1 T) = F(0) \cdot U(z_1 T) + \int_0^T U(z_1 T - \tau) \cdot \frac{\delta F(\tau)}{\delta \tau} d\tau \quad (7)$$

Integration by parts gives :

$$\psi'(z_1 T) = 2\pi \sum_{n=1}^{\infty} \frac{(-1)^n n e^{-\frac{V}{2}}}{\frac{V^2}{4} + n^2 \pi^2} (e^{\frac{V^2}{4}} - e^{-n^2 \pi^2 T}) \sin n\pi z \quad (8)$$

Transforming to the concentration factor gives :

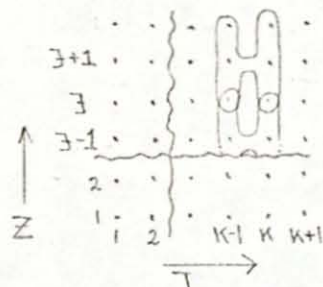
$$\psi(z_1 T) = 2\pi e^{\frac{V}{2}(z-1) - \frac{V^2 T}{4}} \sum_{n=0}^{\infty} \frac{(-1)^n \cdot n}{\frac{V^2}{4} + n^2 \pi^2} (e^{\frac{V^2}{4} T} - e^{-n^2 \pi^2 T}) \sin n\pi z \quad (9)$$

In this case  $\psi$  is varying between -1 and 0. In order to obtain a solution where  $\psi$  varies between 0 and 1, unity is added to the above equation :

$$\psi_{\text{usual}} = 1 + \text{Equation (9)} \quad (10)$$

Numerical solution

The numerical method of solution to be described below may be derived either from finite difference principles (Crandall, 1956) or from variational principles (Zienkiewicz, 1971). The finite difference approach is adopted here (Crandall, p.387). It is convenient to represent the problem as a grid in time and distance.



The terms of equation (4) are modeled as :

$$\frac{\delta^2 \psi^2}{\delta T^2} \approx \frac{\theta(\psi'_{j-1, K+1} - 2\psi'_{j, K+1} + \psi'_{j+1, K+1})}{(\Delta z)^2} + \frac{(1-\theta)(\psi'_{j-1, K} - 2\psi'_{j, K} + \psi'_{j+1, K})}{(\Delta z)^2}$$

$$\frac{\delta \psi'}{\delta T} \approx \frac{\psi'_{j, K+1} - \psi'_{j, K}}{\Delta T}$$

where  $\theta$  is chosen between 0 and 1.

This formulation requires the solution of as many simultaneous equations as there are distance steps for each increment of time factor.

The value of  $\theta$  determines the numerical quality of the formulation, a topic which is extensively discussed by Crandall (1956, p.389). The best choice

of  $\theta$  will depend on the boundary conditions and the grid geometry. For instance, when the concentration factor,  $\psi$ , is kept constant at one boundary, the boundary value of  $\psi'$  will vary exponentially, the exponent being a function of the time and the velocity factors. In order to obtain correct results it is necessary to select  $\theta$  close to 1 for this case, as the increase in  $\psi'$  on the boundary has a large influence on the values at the  $K+1$  time.

#### Users manuals to the computer programs

Three computer programs are available:

- (1) A program computing values of the concentration factor according to Equation (7) of Heiberg (1972). This equation describes the salt concentration in time and distance of a layer with constant initial concentrations and where the concentration at the boundaries are changed simultaneously by an equal amount.
- (2) A program computing values of the concentration factor according to Equation (10) above. This equation describes the salt concentration in time and distance in a layer with constant initial concentration, and where the salt concentration at one boundary is changed by a constant amount, while it is kept at its initial value at the other boundary.
- (3) A program computing values of the concentration factor for arbitrary initial and boundary condition. The program also handles layered sediments where the layers are characterized by different diffusion constants. The computations are based on the finite difference technique described above.

A listing of program (1) is given in Heiberg (1972). A listing of program (2) is shown in Tegn.018. Identical formats of data input is used for the two programs. This is:

First card : Velocity factors (5 values) : 5F10.0

Second card: Time factors (5 values) : 5F10.0

25 isochrones of concentration factors will be computed for each data set. As many data sets as desired may be submitted in one run by repeating cards 1 and 2. The output is essentially as shown in Heiberg (1972).

A listing of program 3 is shown on Tegn. 019-020. The necessary data input is:

Card no.	Symbol	Data	Format
1	HED(20)	Description of the computation	20A4
2	DT	Time increment	
	NTIMIN	Total number of time increments	
	NOZ	Total number of equally spaced distance increments	
	NDIS	Distance increment number above which the diffusion constant is changed	F.10.0, 3I10, F10.0
3	XSIN	Initial value of the concentration factor	
	XSI(1)	Boundary value of the concentration factor at the base	
	XSNDZ	Boundary value of the concentration factor at the top	
	NTZ	Number of different trials where the time increment number when the top concentration becomes zero, is specified	
	NV	Number of different velocity factors to be used	
	NT	Number of different ratios of upper to lower diffusion constants to be used	
		The total number of problems solved is $NTZ \times NV \times NT$	3F10.0, 3F10
4	V(I)	NV different velocity factors	8F10.0
5	T2T1(I)	NT different ratios of upper to lower diffusion constants	8F10.0
6	NTEMP(I)	NTZ different time increment numbers at which the top boundary value of the concentration factor is to be set equal to zero	8I10

An example of the output is shown in Tegn. 021.

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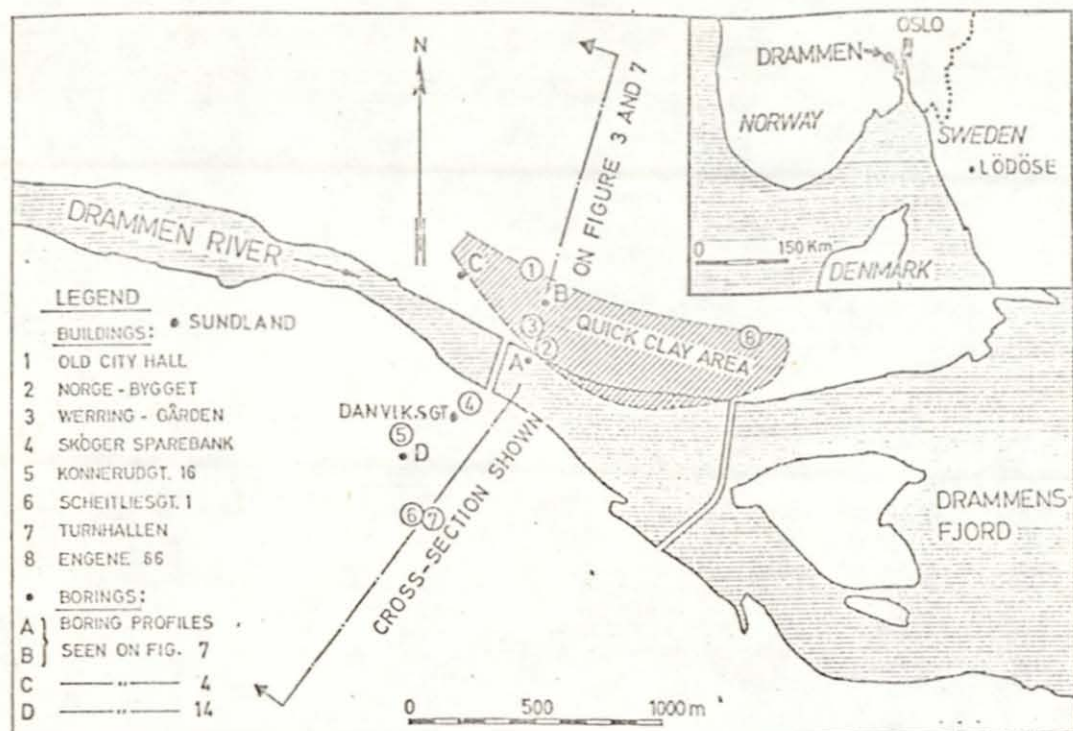


Fig. 1. Map of Drammen showing location of buildings and borings

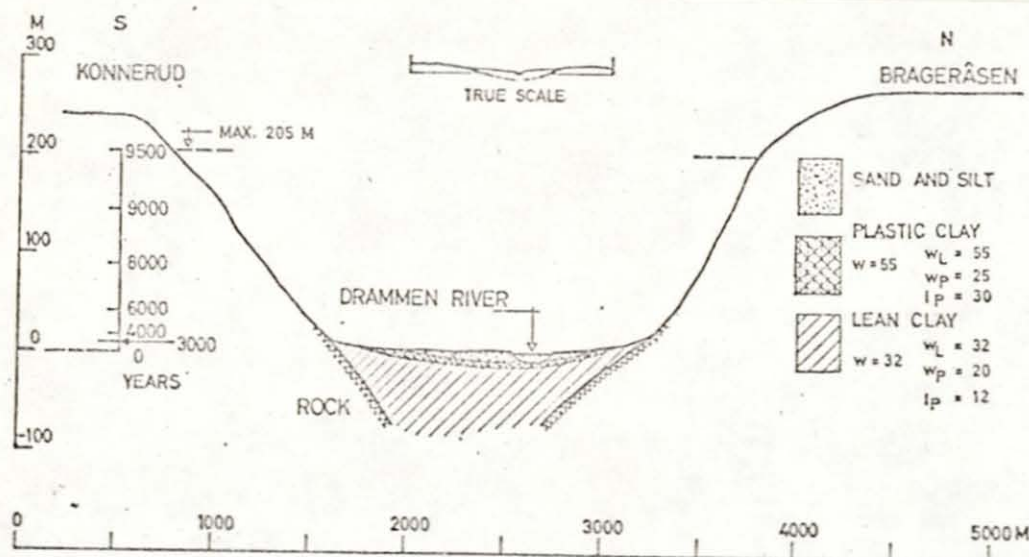
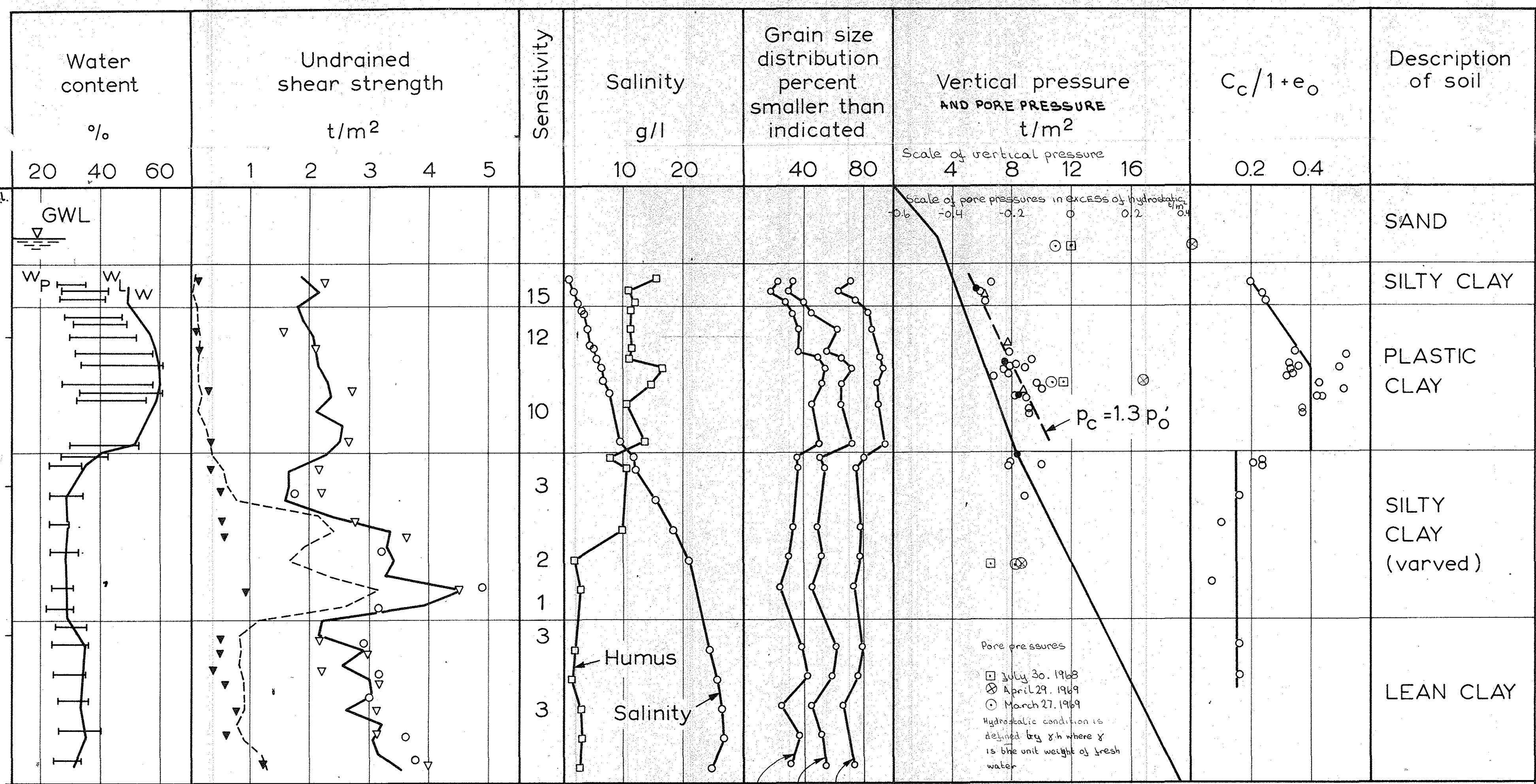


Fig. 3. Cross-section through the valley of Drammen showing the marine sediments and illustrating the land elevation

(from Bjerrum 1967)

	Dato	Tegner SN
Map showing the locations of the Drammen borings.	Godkjent	
	Oppdr. nr.	50700
Norges geotekniske institutt	Tegn. nr.	007



$w_p$  Plastic limit  
 $w_L$  Liquid limit  
 $w$  Water cont.

— Undisturbed } vane test  
 - - - Remoulded }  
 ▽ Undisturbed } cone test  
 ▼ Remoulded }  
 ○ Unconfined compr. test

0 1 2  
 Humus content %

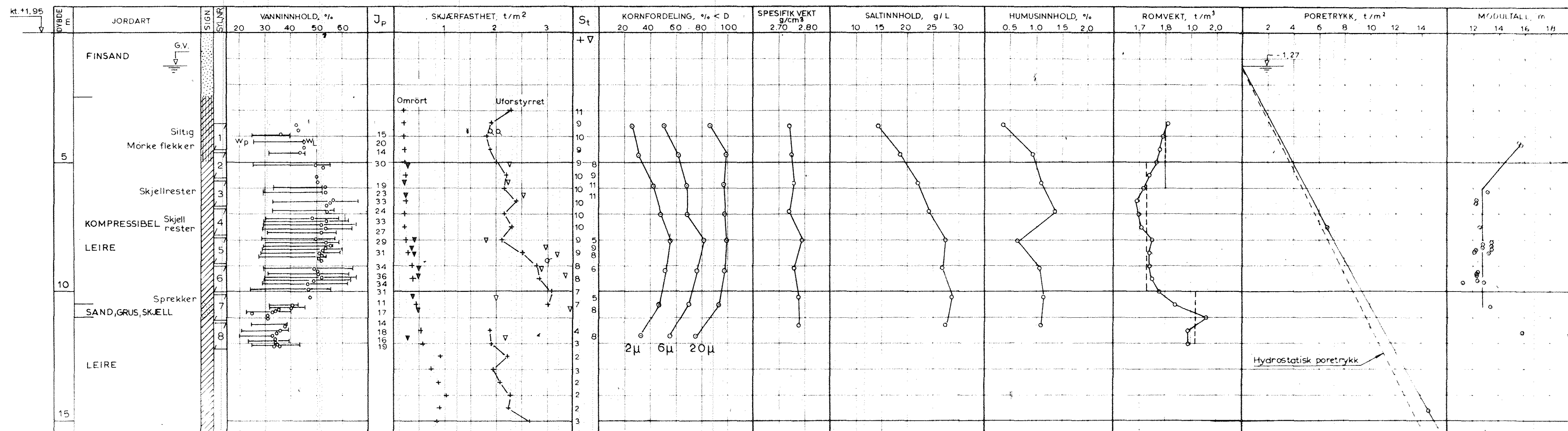
$2\mu$   $6\mu$   $20\mu$

○  $p_c$  from oedometer tests  
 ●  $p_c$  from  $K_0$  triaxial tests  
 △  $p_c$  from direct simple shear tests

Date		Tegner S#	
Godkjent			
Oppdr. nr.		50700	
Tegn. nr.		008	

THE SOIL PROFILE AT SUNDLAND, DRAMMEN  
 Norges geotekniske institutt





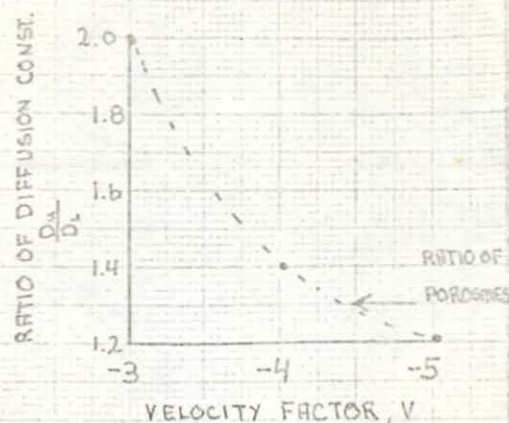
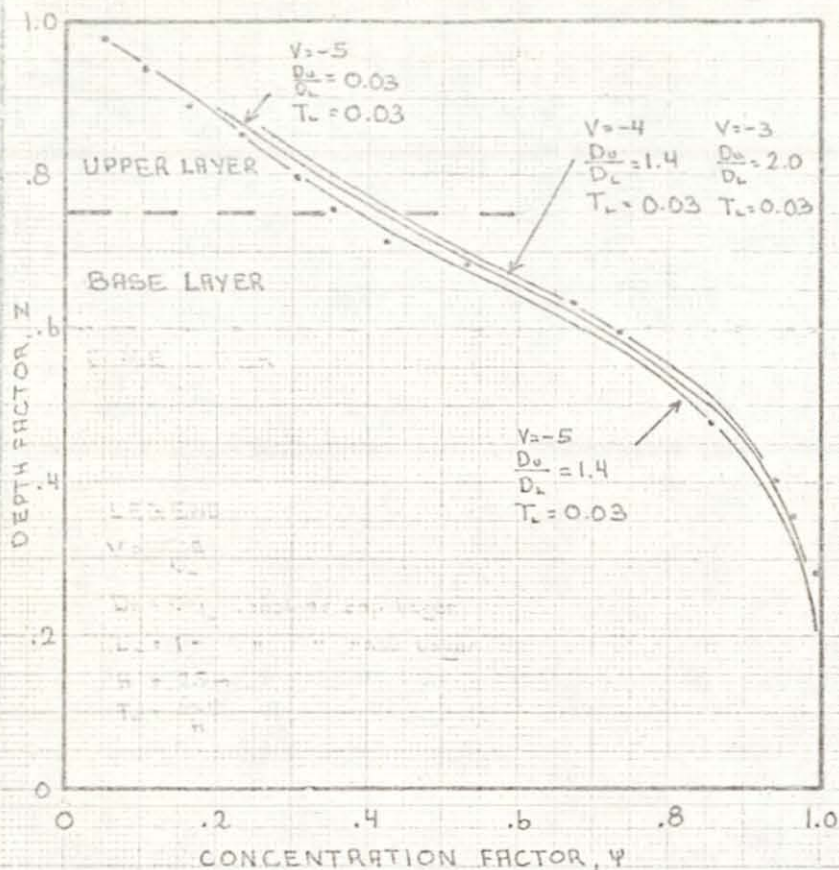
Borprofil  
95 mm prøvetager  
65 x 130 mm vingebor

--- romvekt brukt i beregningene

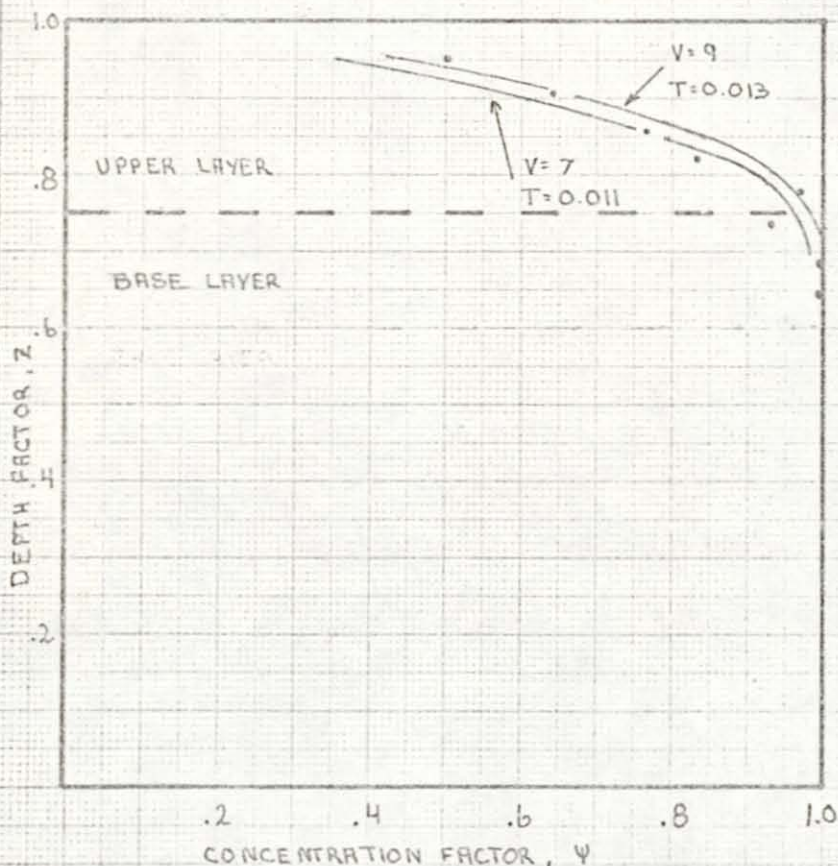
- + vingebor
- o udrenert trykkforsøk
- 15-5 deformasjon ved brudd
- 10
- ∇ uforstyrret konsusforsøk
- ▽ omrørt konsusforsøk

THE SOIL PROFILE AT DAN VIKS GATE, DRAMMEN		Dato	Tegner S H
		Godkjent	
		Oppdr. nr. 50700	
Norges geotekniske institutt		Tegn. nr. 009	

SUNDLUND, DRAMMEN



DANNEVIGSGT. 1b, DRAMMEN



LEGEND

$$V = \frac{U \cdot H}{D_c}$$

$$T = \frac{D_c t}{H^2}$$

$D_u$  = Diff. const. of top layer

$D_c$  = " " " base layer

$U$  = Vertical velocity of flow

$t$  = time

$H$  = 25 m

Interpretations of the Drammen salt profiles.

Date Tegner  
St

Godkjent

Oppdr nr. 50700

Tegn. nr. 010



Map showing the locations of the borings in Central Oslo.

Norges geotekniske Institutt

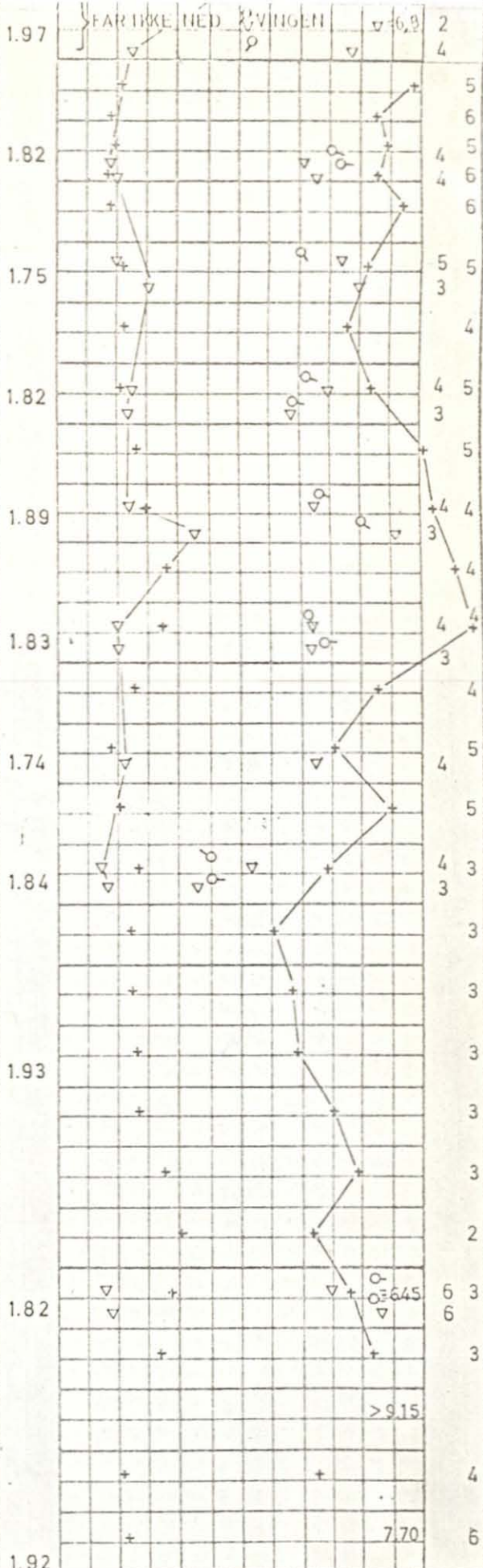
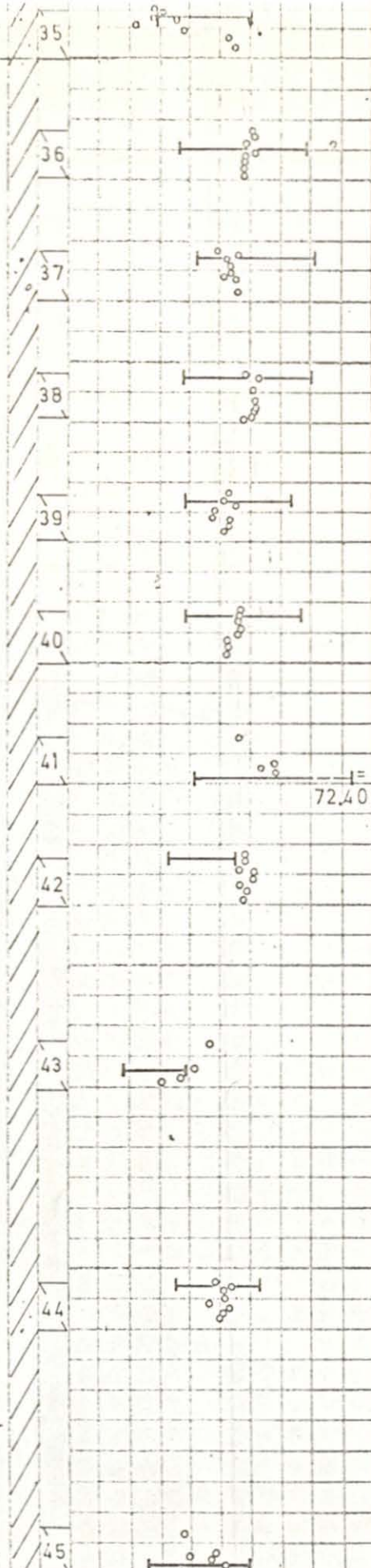
Dato	Tegner
Godkjent	S. W.
Oppdr. nr.	50700
Tegn. nr.	011

10  
15  
20  
25  
30  
35

LEIRE  
MED NOEN SAND  
OG SKJELL -  
RESTER

MERKNAD:  
INNTEGNET VINGEBOR-  
RESULTATER

AVSLUTTET



A/S VATERLAND

BORPROFIL — PRÖVE  $\varnothing$  54 MM.  
HULL NR. 204 ..... NIVÅ +3.69

NORGES GEOTEKNISKE INSTITUTT

- + VINGEBORING
- o ENKELT TRYKKFORSÖK
- $\nabla$  KONUSFORSÖK
- w VANNINNHOLD
- w<sub>p</sub> UTRULLINGSGRENSE
- w<sub>L</sub> FLYTEGRENSE

25.2.69 J.G.S./J.F.  
GODKJ. *BH*  
OPPDRAGSNR.  
**68/62/50700**  
TEGNING NR. **012 003** REV.

100

1000

10000

100

1000

10000



Dato

Tegner  
5H

Godkjent

Profiles of electrical resistivities, Vaterland, Central Oslo.

Oppdr.

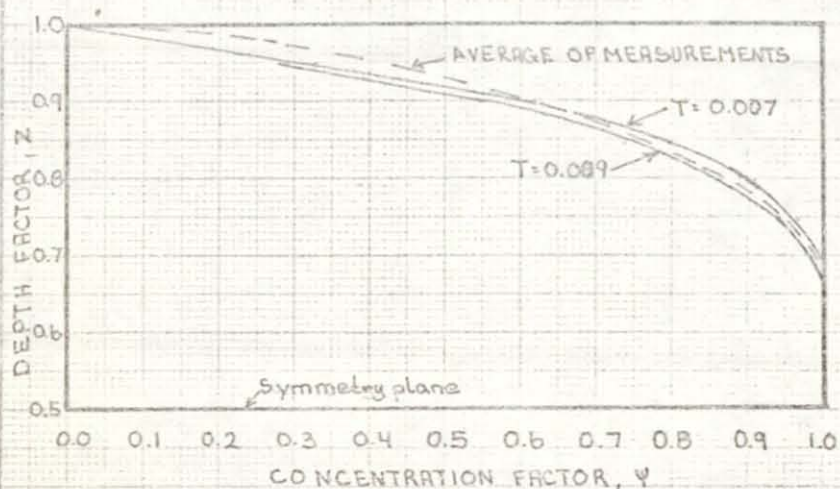
nr. 50700

Norges geotekniske institutt

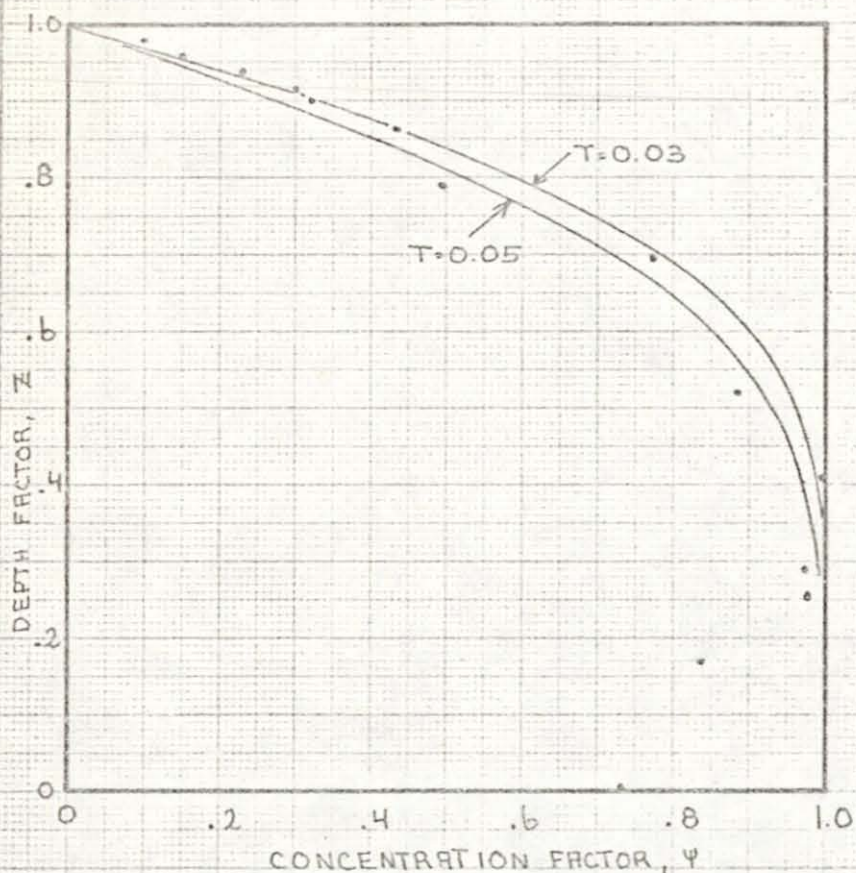
Tegn.  
nr.

013

VATERLAND



STUDENTERLUNDEN



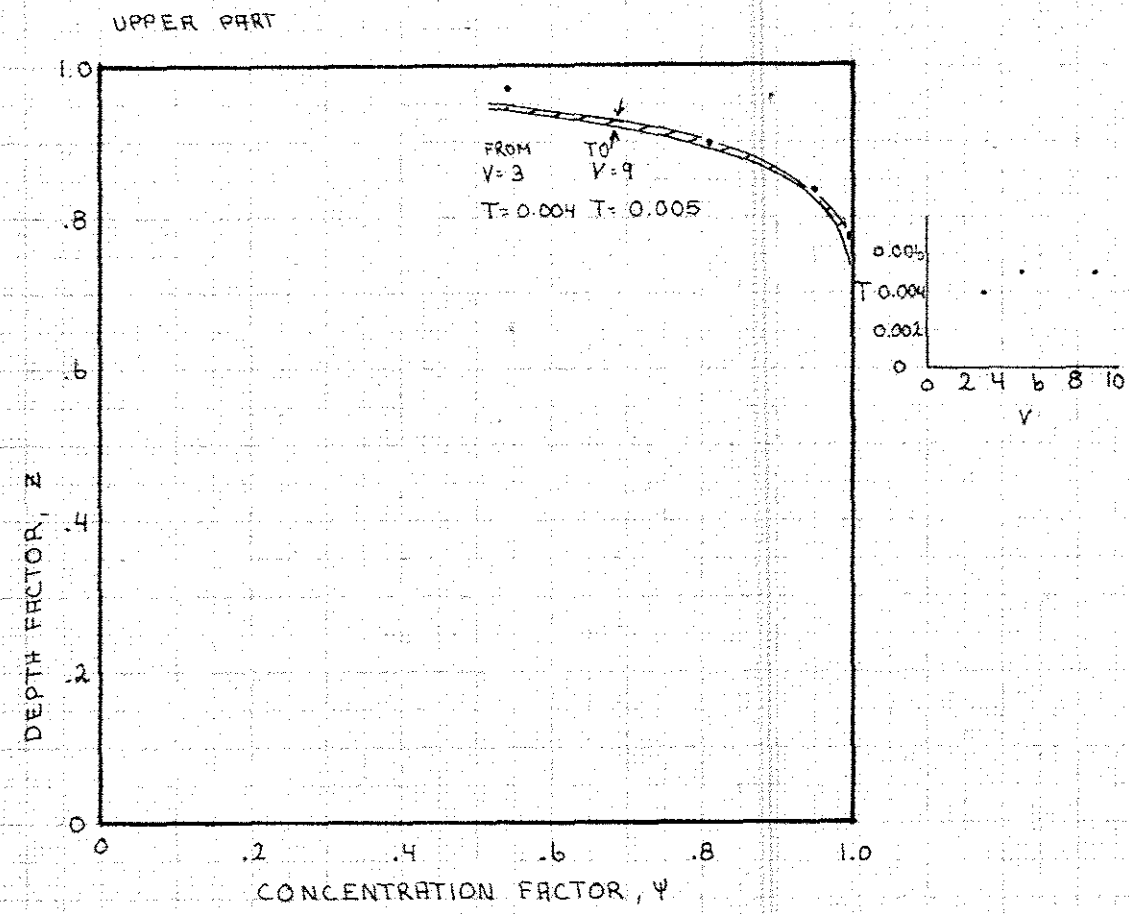
LEGEND

SEE TECH 010  
 H: taken different values

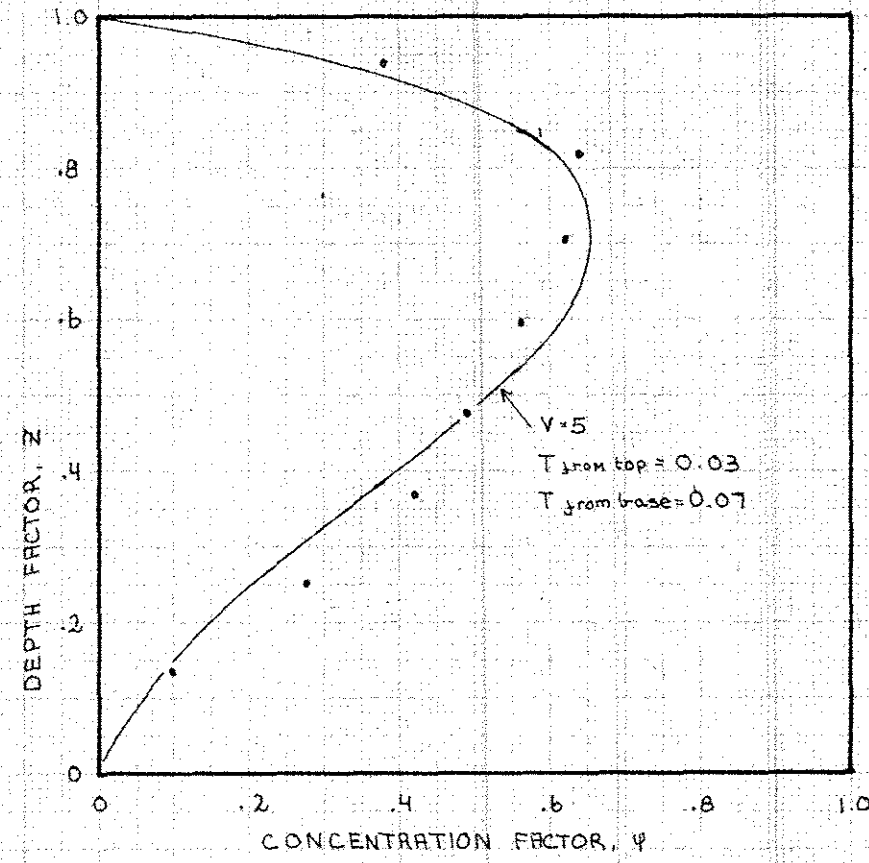
Interpretation of the profiles from Central Oslo.

Dato	Tegner
	98
Godkjent	
Oppdr.	
nr. 50700	
Tegn.	
nr. 014	

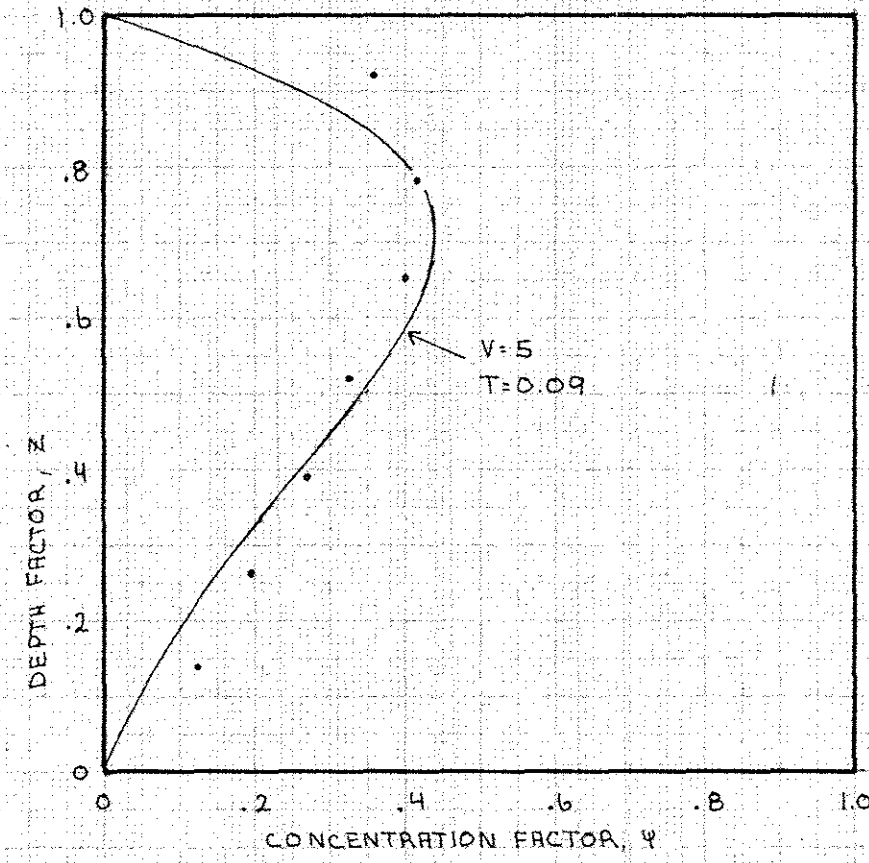
PROFILE 57.7 M. FROM RIVER



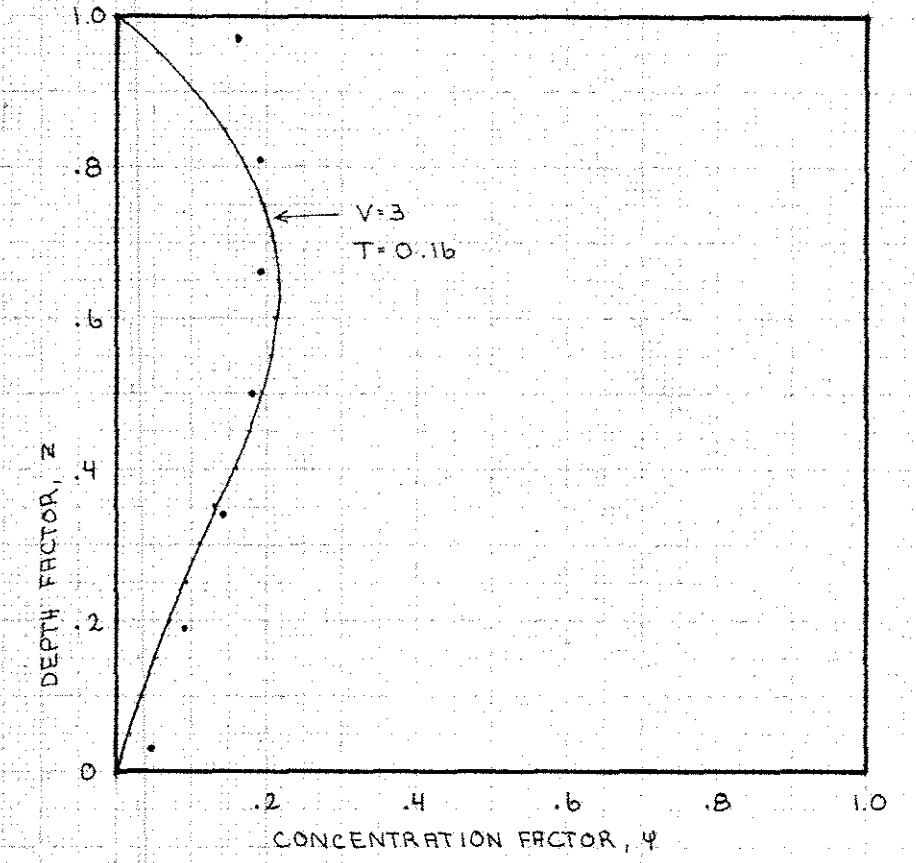
PROFILE 106.8 M. FROM RIVER



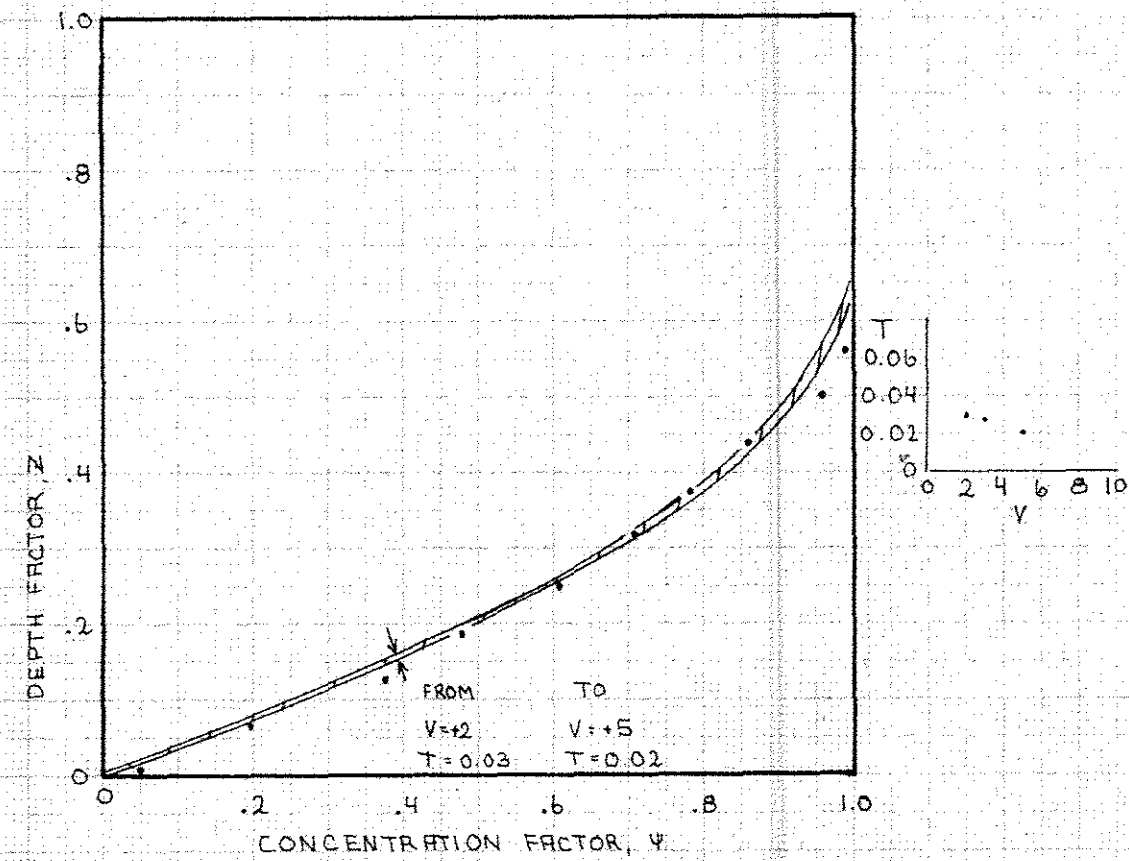
PROFILE 157.6 M. FROM RIVER



PROFILE 180.5 M. FROM RIVER



LOWER PART

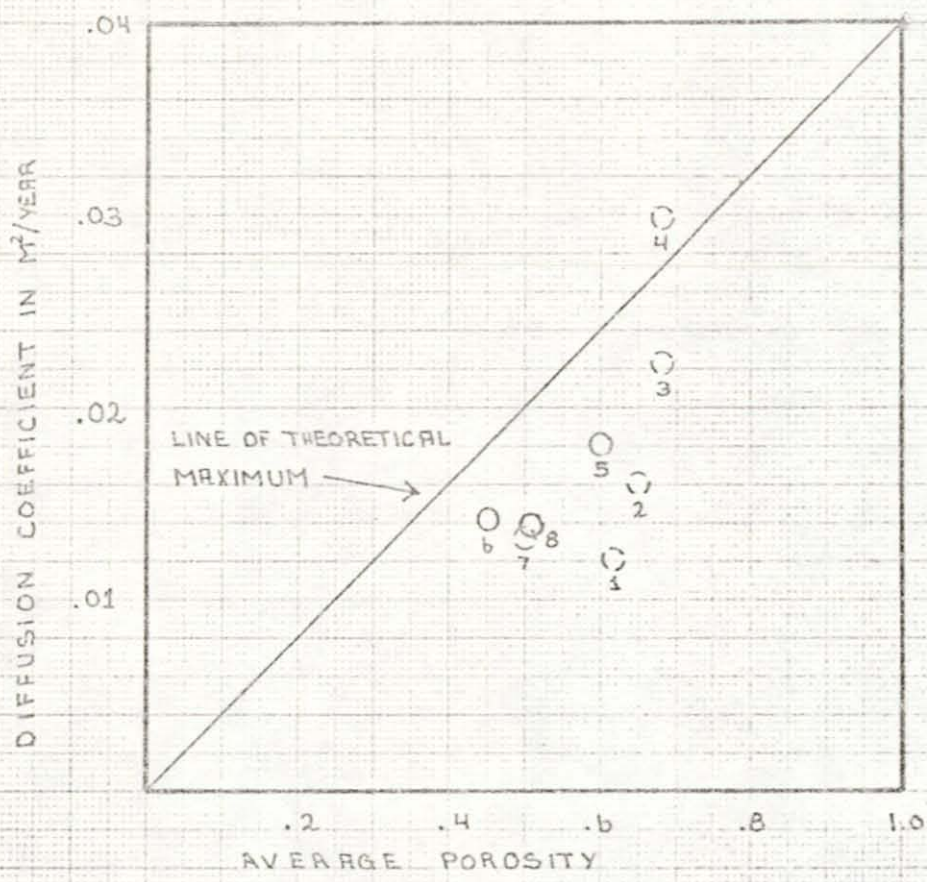


LEGEND  
 See Tegn 010

INTERPRETATION OF THE BORINGS OF ELECTRICAL  
 CONDUCTIVITY, LÖDÖSE, GÖTA RIVER VALLEY, SWEDEN

Norges geotekniska institutt

54  
 50700  
 015



LEGEND

- WELL DOCUMENTED VALUE
- TENTATIVE VALUE

- |          |                         |
|----------|-------------------------|
| 1 LØDØSE | 5 DRAMMEN PLASTIC CLAY  |
| 2 "      | 6 DRAMMEN LEAN CLAY     |
| 3 "      | 7 VATERLAND, OSLO       |
| 4 "      | 8 STUDENTERLUNDEN, OSLO |

Dato Tegner  
SH

The values of diffusion constants derived from the case records as a function of porosity.

Godkjent

Oppdr. nr. 50700



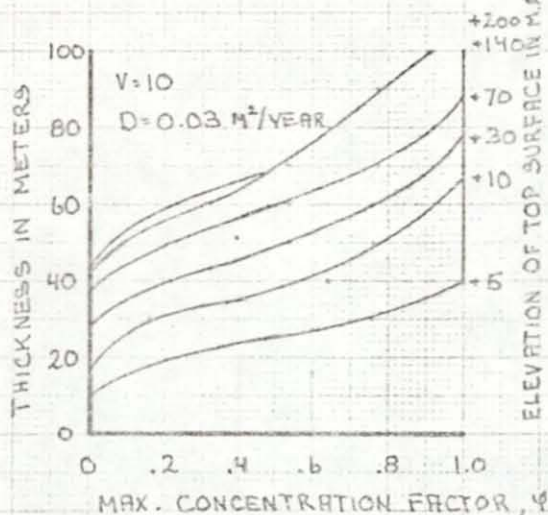
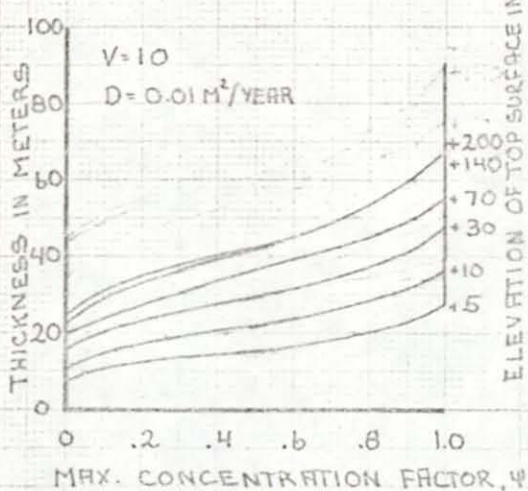
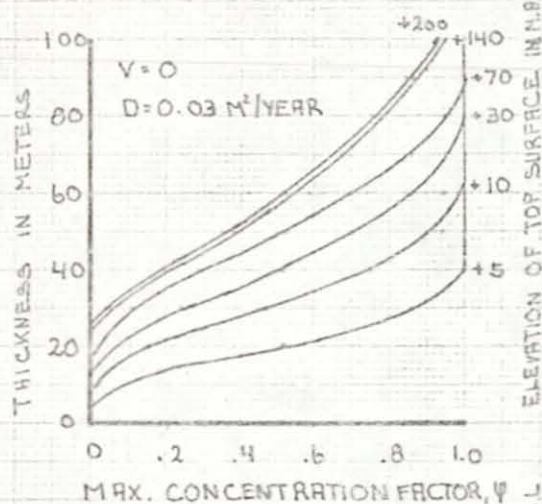
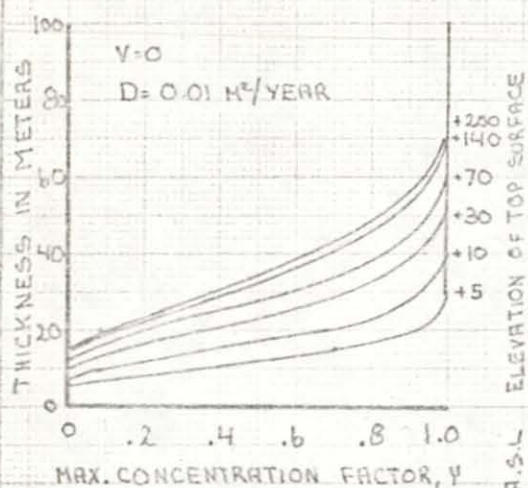
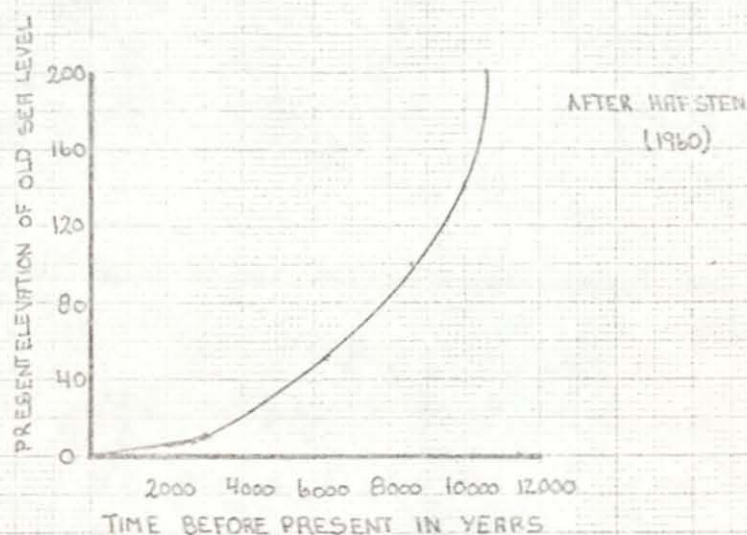
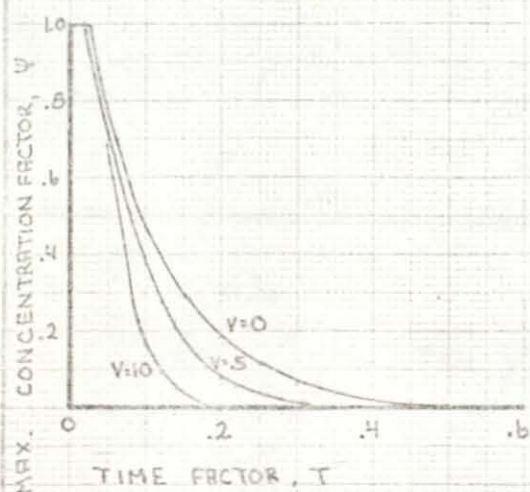


Illustration of a regional interpretation of diffusion  
in the inner Oslofjord area.

Date Tegner

Godkjent

Oppdr. nr. 50700

Tegn. nr. 017

```

DIMENSION Z(20),V(5),T(5),XSI(20,5)
8 WRITE(6,100)
100 FORMAT('1'//10X'SALT CONCENTRATIONS IN BODY ABOVE A SALT RESERVOI
'1R'//)
DO 1 I=1,20
1 Z(I)=0.05*I
READ(5,200)(V(I),I=1,5),(T(I),I=1,5)
200 FORMAT(5F10.0)
PI=3.141593
PIS=PI*PI
DO 2 I=1,5
VS4=V(I)*V(I)/4.
DO 3 J=1,5
VST4=VS4*T(J)
PIST=PIS*T(J)
DO 4 K=1,20
PIZ=PI*Z(K)
POW1=(V(I)*(Z(K)-1.)/2.-VST4
FACT1=2*PI*EXP(POW1)
SUM=0.
DO 5 N=1,200
DEN=VS4+N*N*PIS
FACT2=((-1.)**(N)) *N/DEN
FACT3=EXP(VST4)-EXP(-N*N*PIST)
J1=FACT2*FACT3
SUM=SUM+I1*SIN(N*PIZ)
5 CONTINUE
7 XST(K,J)=1+FACT1*SUM

```

```

4 CONTINUE
3 CONTINUE
WRITE(6,102)(T(ILL),LLL=1,5),(Z(L),V(I),XSI(L,1),XSI(L,2)
1,XSI(L,3),XSI(L,4),XSI(L,5),L=1,19)
102 FORMAT(' Z V T='*E6.2* ' T='*E6.2* ' T='*E6.2*
'1 ' T='*E6.2* ' T='*E6.2*/20(F4.2,E7.2,5E10.3//)
2 CONTINUE
GO TO 8
END

```

	Dato	Tegner Stt
Listing of computer program calculating the salt concentration in a uniform layer where the salt concentration at the top boundary is changed.	Godkjent	
	Oppdr. nr.	50700
Norges geotekniske institutt	Tegn. nr.	018

```

C PROGRAM PERFORMING AN IMPLICIT FINITE DIFFERENCE CALCULATION OF
C THE TRANSPORTATION OF SALT IN A POROUS MEDIUM WITH A MOVING PORE
C FLUID
DIMENSION XSI(30),HED(20),XSIM(30,2),V(5),T2T1(5)
DIMENSION NTEMP(5)
C INPUT NECESSARY DATA
9 READ(5,100) HED
READ(5,101) DT,NTIMIN,NDZ,NDIS
NDZ1=NDZ+1
DZ=1./NDZ
READ(5,103)XSIN,XSI(1),XSNDZ,NTZ,NV,NT
READ(5,102)(V(I),I=1,NV)
READ(5,102)(T2T1(I),I=1,NT)
THETA=0.9
READ(5,1000)(NTEMP(I),I=1,NTZ)
1000 FORMAT(8I10)
DO 10 ITEMP=1,NTZ
NTU=NTEMP(ITEMP)
DO 10 IV=1,NV
DO 11 IT=1,NT
XSI(NDZ1)=XSNDZ
WRITE(6,200) HED
WRITE(6,204)NV,NT,V(IV),DT,NTIMIN,NDZ,NDIS,T2T1(IT),XSIN,
1 XSI(1),XSI(NDZ1),NTU
C ESTABLISH AN INITIAL CONDITTON

```

```

Z=0.
T=0.
XSIM(1,1)=XSI(1)*EXP(-V(1)*Z/2.)
XSIM(NDZ1,1)=XSI(NDZ1)*EXP(-V(IV)*Z/2.)
DO 1 I=2,NDZ
Z=Z+DZ
XSI(I)=XSIN
XSIM(I,1)=EXP(-V(IV)*Z/2.)*XSI(I)
1 CONTINUE
C OUTPUT HEADING AND INITIAL CONDITIONS
WRITE(6,202)(I,T=1,30),T,(XSI(I),I=1,NDZ1)
L=1
N=2
DO 8 K=1,NTIMIN
T=T+DT
XSIM(NDZ1,N)=XSI(NDZ1)*EXP(V(IV)*V(IV)*T/4.-V(IV)/2.)
XSIM(1,N)=XSI(1)*EXP(V(IV)*V(IV)*T/4.)
IF(K.GE.NTU) XSIM(NDZ1,N)=0.
C SOLVE FOR XSIM USING GAUSS SEIDEL ITERATION
DO 4 I=1,200
DIF=0.
DO 5 J=2,NDZ
DENT=DT
IF(J.EQ.NDIS+1)DENT=DT*(T2T1(IT)+1)/2.
IF(J.GT.NDIS+1) DENT=T2T1(IT)*DT
XSIMJ=XSIM(J,N)
DEN=2*THETA/(DZ*DZ)+1./DENT
TERM1=(1.-THETA)*(XSIM(J-1,L)-2.*XSIM(J,L)+XSIM(J+1,L))/(DZ*DZ)
1 +XSIM(J,L)/DENT
TERM2=THETA*(XSIM(J-1,N)+XSIM(J+1,N))/(DZ*DZ)
XSIM(J,N)=(TERM1+TERM2)/DEN

```

	Dato	Tegner Stt
Listing of a general computer program computing salt concentrations based on an implicit finite difference scheme.	Godkjent	
	Oppdr. nr.	50700
Norges geotekniske institutt	Tegn. nr.	019

```

5 CONTINUE
IF (ABS(DIF) .LT. 1.E-6) GO TO 6
4 CONTINUE
WRITE(6,201)
C OUTPUT THE LAST COMPUTED CONCENTRATIONS
6 DO 7 J=1,NDZ1
Z=DZ*(J-1)
XST(J)=XSIM(J,N)*EXP((V(IV)*Z*2.-V(IV)+V(IV)*T)/4.)
7 CONTINUE
WRITE(6,203) T,(XST(J),J=1,NDZ1)
ITT=L
L=N
N=ITT
8 CONTINUE
11 CONTINUE
10 CONTINUE
GO TO 9
100 FORMAT(20A4)
101 FORMAT(F10.0,3T10,F10.0)
102 FORMAT(8F10.0)
103 FORMAT(3F10.0,3T10)
200 FORMAT('1',20A4//)
201 FORMAT(' ITERATION FAILED TO CONVERGE IN 100 TRIALS')
202 FORMAT(' .4HTIME,30(1X,I2,1X)//F5.3,(30F4.2)')
203 FORMAT(F5.3,(30F4.2))
204 FORMAT(' RUN INFORMATION :'/1H,15(1H-)//
1      * NUMBER OF VELOCITY FACTORS ',27(1H.)2H :',T5/
2      * NUMBER OF RATIOS OF UPPER TO LOWER DIFFUSION CONSTANTS :
3      * I5//
4      * GENERAL INFORMATION :'/1H ,19(1H-)//
5      17H VELOCITY FACTOR ,36('.'')2H :',F10.3/

1      17H TIME INCREMENT .,36('.'')2H :',F10.3/
2      27H NUMBER OF TIME INCREMENTS ,26('.'')2H :',I10/
3      31H NUMBER OF VERTICAL INCREMENTS ,22('.'')2H :',I10/
4      55H VERTICAL INCREMENT NUMBER WHERE DISCONTINUITY OCCURS :
5      110/
6      44H RATIO OF UPPER TO LOWER DIFFUSION CONSTANT ,9('.'')2H
7      F10.3//
*      * BOUNDARY CONDITIONS :'/1H,19(1H-)//
*      * INITIAL VALUE OF CONCENTRATION ',41(1H.)2H :',F10.3/
*      * BASE VALUE OF CONCENTRATION ',44(1H.)2H :',F10.3/
*      * TOP VALUE OF CONCENTRATION ',45(1H.)2H :',F10.3/
*      * TIME INCREMENT NUMBER WHERE THE TOP VALUE OF CONCENTRATI
*N BECOMES ZERO :',I10//)
END

```

	Dato	Tegner SH
Tegn. 019 - continued	Godkjent	
	Oppdr. nr. 50700	
Norges geotekniske institutt	Tegn. nr.	020

LOOSE PROFILES 180-106. PARAMETER STUDY

RUN INFORMATION :

-----  
 NUMBER OF VELOCITY FACTORS ..... : 5  
 NUMBER OF RATIOS OF UPPER TO LOWER DIFFUSION CONSTANTS : 1

GENERAL INFORMATION :

-----  
 VELOCITY FACTOR ..... : 1.000  
 TIME INCREMENT ..... : .010  
 NUMBER OF TIME INCREMENTS ..... : 10  
 NUMBER OF VERTICAL INCREMENTS ..... : 20  
 VERTICAL INCREMENT NUMBER WHERE DISCONTINUITY OCCURS : 1  
 RATIO OF UPPER TO LOWER DIFFUSION CONSTANT ..... : 1.000

BOUNDARY CONDITIONS :

-----  
 INITIAL VALUE OF CONCENTRATION ..... : 1.000  
 BASE VALUE OF CONCENTRATION ..... : .000  
 TOP VALUE OF CONCENTRATION ..... : 1.000  
 TIME INCREMENT NUMBER WHERE THE TOP VALUE OF CONCENTRATION BECOMES ZERO : 1

TIME	VERTICAL INCREMENT NUMBERS																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
.000	.00	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.00
.010	.00	.32	.59	.75	.85	.91	.94	.96	.98	.98	.99	.99	.99	.98	.96	.94	.90	.82	.69	.46	.00	
.020	.00	.21	.40	.56	.69	.78	.85	.90	.93	.95	.96	.96	.95	.93	.90	.85	.77	.65	.49	.26	.00	
.030	.00	.16	.31	.45	.58	.68	.76	.82	.86	.89	.90	.91	.89	.87	.82	.76	.66	.54	.39	.21	.00	
.040	.00	.13	.26	.38	.50	.59	.68	.74	.79	.82	.84	.84	.83	.80	.75	.68	.59	.47	.33	.17	.00	
.050	.00	.11	.22	.33	.43	.53	.60	.67	.72	.76	.78	.78	.77	.73	.69	.62	.53	.42	.30	.15	.00	
.060	.00	.10	.20	.29	.38	.47	.54	.61	.65	.69	.71	.71	.70	.67	.63	.56	.48	.38	.27	.14	.00	
.070	.00	.09	.17	.26	.34	.42	.49	.55	.59	.63	.65	.65	.64	.61	.57	.51	.43	.34	.24	.12	.00	
.080	.00	.08	.16	.23	.31	.38	.44	.49	.54	.57	.59	.59	.58	.56	.52	.46	.40	.31	.22	.11	.00	
.090	.00	.07	.14	.21	.28	.34	.40	.45	.49	.52	.53	.54	.53	.51	.47	.42	.36	.28	.20	.10	.00	
.100	.00	.06	.13	.19	.25	.31	.36	.41	.44	.47	.49	.49	.48	.46	.43	.38	.33	.26	.18	.09	.00	

Typical output of the program given on Tegn. 019-020.

Norges geotekniske institutt

Dato	Godkjent	Tegner
		SH
Oppdr. nr.	50700	Tegn. nr.
		021